Control strategies for network efficiency and resilience with route choice

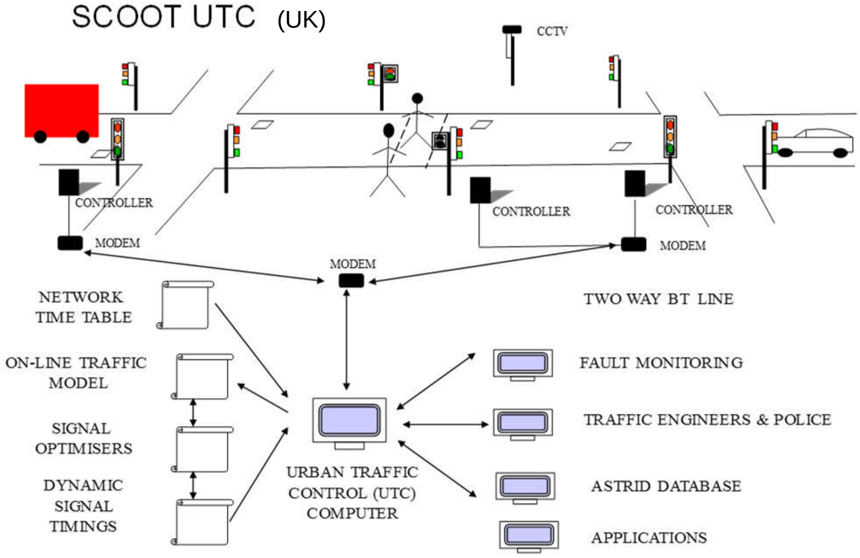
Andy Chow
Rui Sha
Centre for Transport Studies
University College London, UK

Centre for Transport Studies

UCL ENGINEERING
Change the world

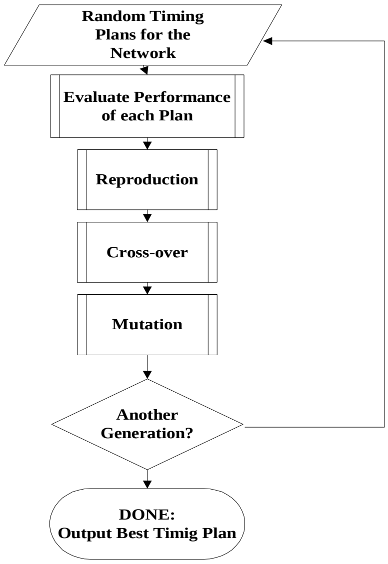
Centralised strategies

SCOOT UTC (UK)



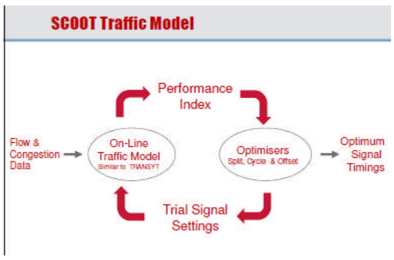
Centralised strategies

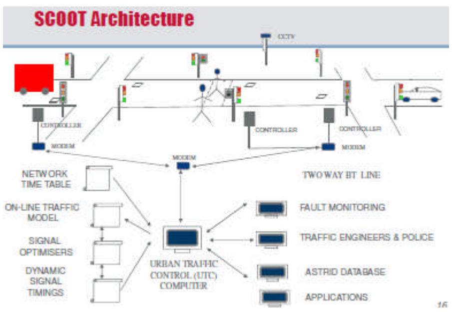
- **Some effective centralised solution approaches:**
- ‘**Genetic algorithm’** solution – a probabilistic heuristics (offline) that mimics the natural selection process for obtaining global optimal control plan

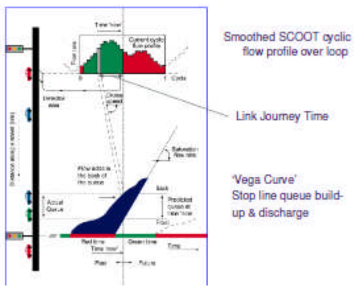


Centralised strategies

- ‘**Hill-climbing algorithm’ (e.g. in SCOOT)** solution – an online heuristics making gradual adjustments on timing plans w.r.t. real time traffic







Linear Quadratic Regulator (TUC)

- Formulation:

$$\min_{\mathbf{u}} Z = \sum_{k=0}^{K} \left(\mathbf{x}_k^T \mathbf{Q} \mathbf{x}_k + \mathbf{u}_k^T \mathbf{R} \mathbf{u}_k \right)$$

k – cycle index;

$\mathbf{x_k}$ – (= $[x_i(k)]$) queues on all links *i* by the end of each cycle *k*
(state variable)

$\mathbf{u_k}$ – (= $[u_i(k)]$) adjustment made on green splits in cycle *k*
(control variable)

$$\mathbf{u}_k = \mathbf{g}_k - \mathbf{g}^N$$

Linear Quadratic Regulator

- The objective function is subject to the state equation for all *i*

$$x_i(k+1) = x_i(k) + C\left[d_i(k) - s_i(k)\right]$$

- For source links:

$$x_i(k+1) = x_i(k) + C\left[d_i(k) - s_i \frac{g_i(k)}{C}\right]$$

- For intermediate links:

$$x_i(k+1) = x_i(k) + C\left[\sum_{\forall j \in J(i)} \beta_{ji} s_j \frac{g_j(k)}{C} - s_i \frac{g_i(k)}{C}\right]$$

where *J*(*i*) is the set of links upstream of *i* ;
β_{ji} is the proportion of flow in *j* flowing into *i*

Linear Quadratic Regulator

- The state equations can be summarised as

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{B}\mathbf{g}_k + C\mathbf{d}_k$$

for some appropriate matrix **B** (a 'sparse' matrix of 'minus' saturation flows for each link)

- We can derive the feedback control rule as the optimality condition (setting **d** = 0; $k \to \infty$) of the control problem as:

$$\mathrm{g}(k) = \mathrm{g}^N - \mathbf{L}\mathrm{x}(k)$$

where the gain matrix **L** can be derived through solving the corresponding Bellman equation

Linear Quadratic Regulator

- The gain matrix can be determined as:

$$\mathbf{L} = -(\mathbf{R} + \mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{P}$$

- Where **P** can be solved (iteratively) from the following Riccati equation

$$\mathbf{P} = \mathbf{S} + \mathbf{P} - \mathbf{P}\mathbf{B}(\mathbf{R} + \mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{P}$$

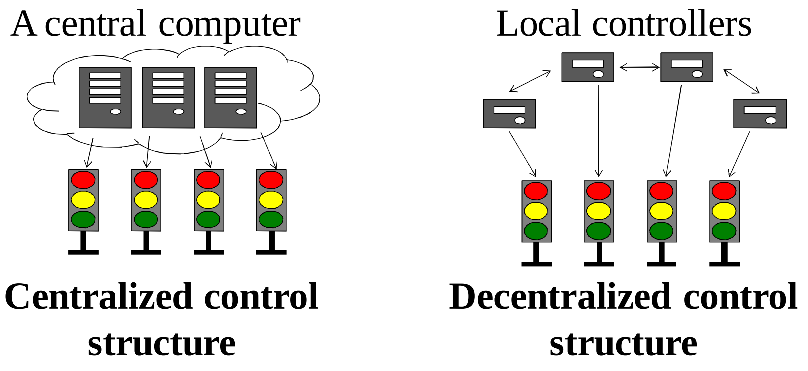
- Note that **L** is generally non-diagonal (as a centralised regulator) but sparse matrix (with most elements zero)

$$\mathrm{g}(k) = \mathrm{g}^N - \mathbf{L}\mathrm{x}(k)$$

Centralised strategies

- Derive control plans with consideration of the entire system for global objective (e.g. lowest system-wide delay)
- Improve global efficiency, while it may come at the expense of computational effort, and communication links…
- Centralised design may(?) also cause the underlying system less robust in case of incidents (e.g. see Helbing, Le, etc)

Control architecture



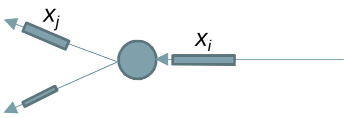
UCL

Max-pressure controller (distributed)

- Define ‘pressure’ on phase *i* as a function of queue sizes

$$w_i(k) = Q_i \left[x_i(k) - \sum_{j=1}^{l_{out}} \beta_{ij} x_j(k) \right],$$

where Q_i is the saturation flow on *I*



- ‘Right-of-way’ is assigned to phase has the maximum pressure

Reference: Varaiya, P (2013) Max pressure control of a network of signalized intersections, Transportation Research Part C, 36, 177-195.

UCL

Max-pressure controller (distributed control)

- Feedback control on queue sizes:

$$\mathbf{g}(k) = \mathbf{g}^N - \mathbf{L}\ \mathbf{w(k)} \qquad w_i(k) = Q_i \left[x_i(k) - \sum_{j=1}^{l_{out}} \beta_{ij} x_j(k) \right],$$

while:

L is diagonal (the control system is distributed)

x refers to ‘pressure’ at the junction

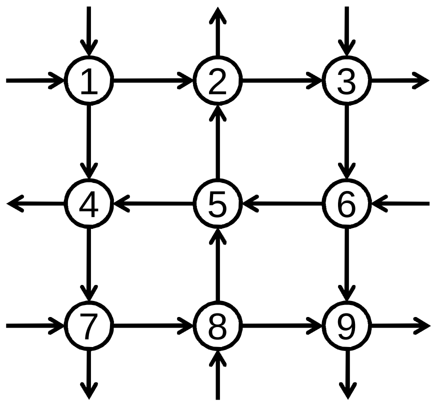
(which is defined as a difference between the queues measured at upstream and <u>downstream</u> of the signal)

Reference: Varaiya, P (2013) Max pressure control of a network of signalized intersections, Transportation Research Part C, 36, 177-195.

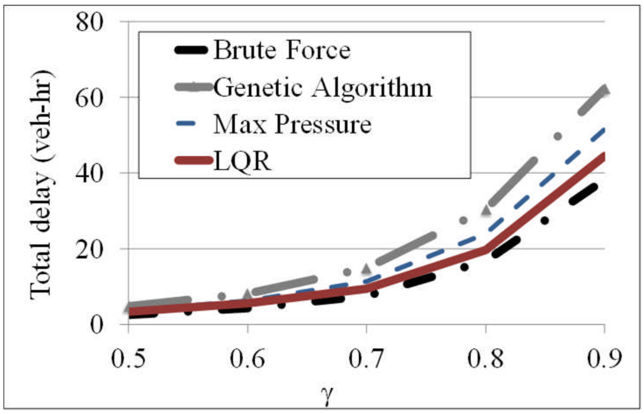
Inflows (with 10% c.o.v.):

$$\lambda_x(t) = \begin{cases} 0.6\mu, & 0 \le t \le 300 \\ \mu, & 301 \le t \le 600 \\ 0.6\mu, & 601 \le t \le 900 \\ 0, & t > 900 \end{cases}$$

$$\lambda_y(t) = \begin{cases} 0.48\mu, & 0 \le t \le 300 \\ 0.8\mu, & 301 \le t \le 600 \\ 0.48\mu, & 601 \le t \le 900 \\ 0, & t > 900 \end{cases}$$



Turning ratio at all nodes: 30%
Traffic modelled by CTM



$$\gamma = \frac{1}{IT} \sum_{t=1}^{T} \sum_{i=1}^{I} \frac{f_i(t)}{Q_i(t)}.$$

UCL

Experiment on a micro-platform

- How about on a microscopic platform with route choice?
- If incident(s) occur?

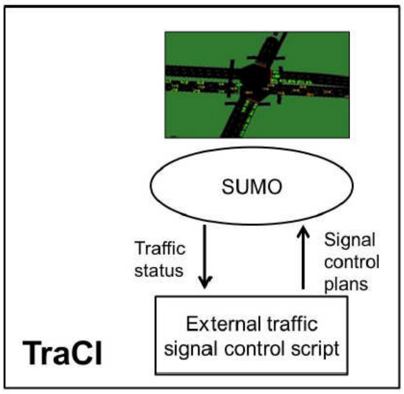


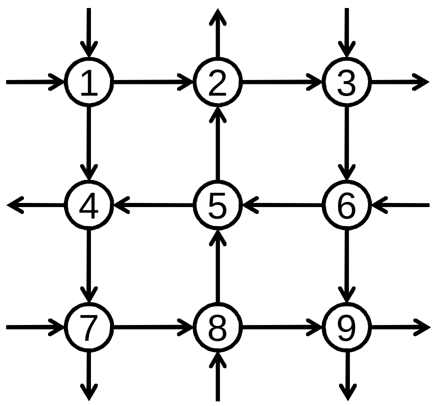
Figure 5.1: Traffic control interface of SUMO

UCL

Inflows (with 10% c.o.v.):

$$\lambda_x(t) = \begin{cases} 0.6\mu, & 0 \leq t \leq 300 \\ \mu, & 301 \leq t \leq 600 \\ 0.6\mu, & 601 \leq t \leq 900 \\ 0, & t > 900 \end{cases}$$

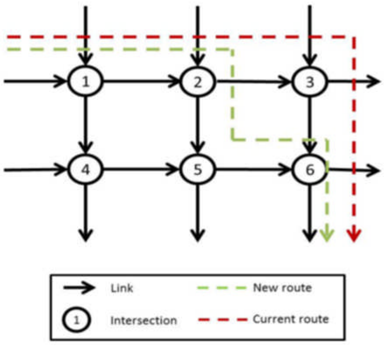
$$\lambda_y(t) = \begin{cases} 0.48\mu, & 0 \leq t \leq 300 \\ 0.8\mu, & 301 \leq t \leq 600 \\ 0.48\mu, & 601 \leq t \leq 900 \\ 0, & t > 900 \end{cases}$$



Turning ratios vary **w.r.t. traffic condition**
Traffic modelled by **SUMO**

Routing algorithm (iterative shortest path procedure)

- Construct an origin-destination matrix following the previous demand setting
- Each vehicle departs from its origin, proceeds toward the destination along the prevailing shortest path
- The path will be revised whenever the vehicle reaches node based upon prevailing traffic conditions (queues / travel times)



Cyclic Max-pressure (Backpressure)

- Given the 'pressure' function:

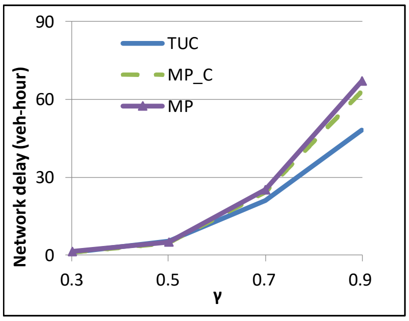
$$w_i(k) = Q_i \left[x_i(k) - \sum_{j=1}^{I_{out}} \beta_{ij} x_j(k) \right],$$

Green splits are every cycle proportionally as:

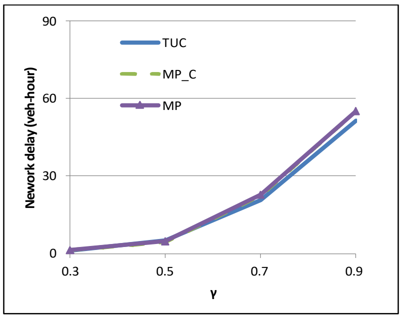
$$V_i(k) = \frac{\exp\{\eta w_i(k)\}}{\sum_{j=1}^{I_{in}} \exp\{\eta w_j(k)\}},$$

Reference: Le, et al. (2015) Decentralised signal control for urban road networks, Transportation Research Part C, 58, 431-450.

With vs without re-routing



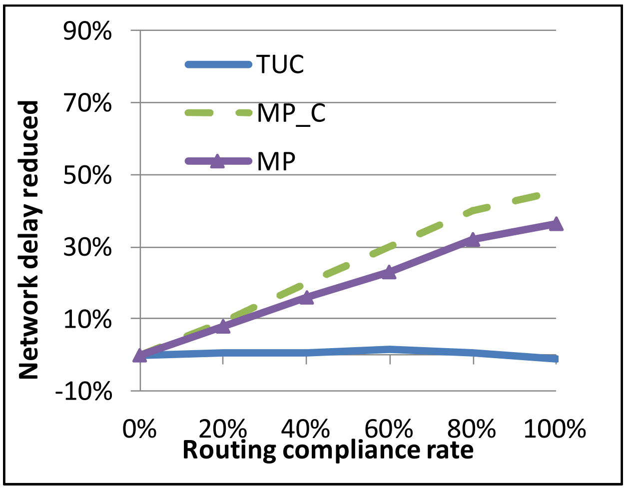
Without re-routing



With re-routing

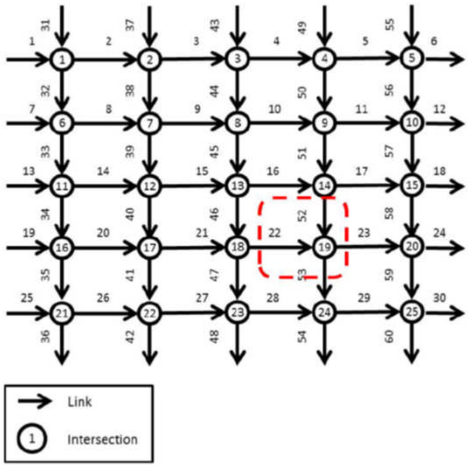
Effect of re-routing rate

- Assume some vehicles would not re-route regardless of the prevailing traffic conditions…

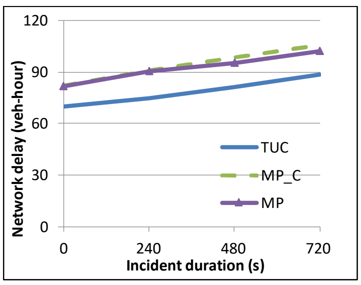


An incident occurs …

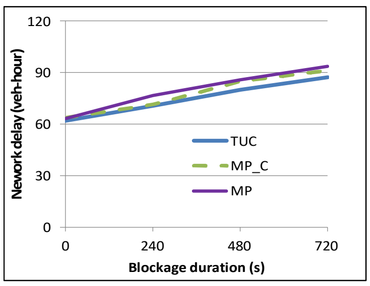
- Suppose node 19 (and hence links 22 and 52) is down …



With vs without re-routing



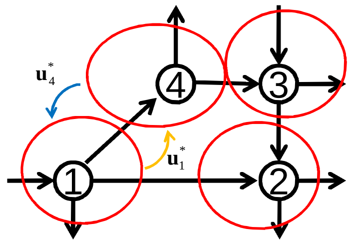
Without re-routing



With re-routing

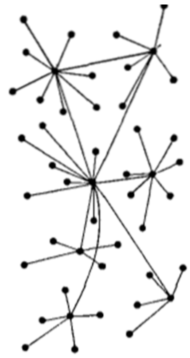
UCL

Decentralised MPC …



Examples of methods:

- Multi-agent MPC (de Oliveira and Camponogara, 2010);
- Alternating directions method of multipliers (ADMM, Reilly and Bayen, 2015);
- Approximation with Principle Component Analysis (PCA, Rinaldi, et al, 2016) …



UCL

Concluding remarks

- A performance comparison of centralised and distributed control for urban road networks
- Significance of re-routing
  - Could be due to the setting of TUC …
- Consideration of incidents (resilience)
  - Centralisation / coordination is needed
- Ongoing work:
  - Decentralisation / decomposition
  - Online solution algorithm
    (with consideration of travel behaviour changes)